Exercises

1. Consider the basic unilateral accident model where $p(x)$ is the probability of an accident, $p'<0$, $p''>0$; x is the injurer's spending on care; and D is the victim's *fixed* damages. Under a strict liability rule, the injurer faces liability, L, equal to D, and hence chooses efficient care of x^*.

(a) Suppose that the injurer has limited assets, A, to spend on liability. That is, $L \leq A$. How does this constraint affect the injurer's care choice when $A<D$? Show that care is increasing in A over this range.

(b) Now suppose that care involves an out-of-pocket expenditure, so that the asset constraint is now $x+L \leq A$. Show that in this case, care may be greater than or less than x^*. Explain the result intuitively.

2. In the standard accident model, the risk of accidents often depends on the parties' activity levels as well as their care. Consider a model where accident risk depends on the *injurer's care* and the *victim's activity level*. For example, a pedestrian decides how often to walk on a busy street, and a driver decides how fast to drive. Let:

> z = victim's activity level;
> $B(z)$ = gross benefit of z to the victim, where $B'>0$, $B''<0$;
> x = injurer's spending on care;
> $L(x)$ = victim's expected damage per unit of z, where $L'<0$, $L''>0$.

(a) Derive the first order conditions describing the optimal activity and care levels.

(b) Show the actual choices of z and x under *no liability*, *strict liability*, and *negligence*. (Assume that the due standard under negligence is set at the efficient care level from part (a).) How do they compare to the social optimum?

3. Consider an accident setting in which an injurer can take care of x to reduce the probability of an accident, $p(x)$ ($p'<0$, $p''>0$). If an accident occurs, the victim's loss is $D(y)$, where y represents an expenditure made after the fact to *mitigate* the damages. Either the victim or the injurer can invest y.

(a) Derive the first order conditions characterizing the optimal levels of x and y. (Note that the optimum does not depend on which party chooses y.)

(b) Suppose initially that the injurer chooses both x and y. Derive the conditions describing the injurer's actual choice of these variables under *strict liability* and under *negligence* (where due care is set at the socially optimal x). Which rule is preferred?

(c) Now suppose that the victim chooses y while the injurer continues to choose x. Again, derive the actual choices of these variables under strict liability and negligence. Which rule is preferred in this case?

4. Suppose that the gross demand for a risk product is given by $10-q$, and the producer's total cost of production is $q^2/2$, where q is quantity. Also suppose that the product creates a per unit risk of an accident equal to p, the damages per accident are D, and the producer's share of liability is s, $0 \leq s \leq 1$.

(a) Derive the efficient level of production of the product and show that the competitive market achieves that outcome regardless of s.

(b) Now suppose that the consumer perceives the risk of an accident to be αp, where $\alpha \neq 1$. Derive the market equilibrium as a function of s and α and show that q is only efficient if $s=1$ (i.e., if the producer is strictly liable). Also show that the sign of $\partial q/\partial s$ depends on whether α is less than or greater than one. Provide intuition for your answers.

5. Consider a contract between a buyer and a seller, where:

V = value of performance to the buyer;
C = cost of performance to the seller (a random variable);
P = contract price, payable on performance.

Suppose that the realized value of C was unknown at the time the contract was made, but at the date of performance it turns out that $C>P$.

(a) Show that if bargaining between the buyer and seller is costless, the parties will agree on a price increase (or bonus) b such that performance occurs if and only if performance is efficient at the realized value of C.

(b) Derive the upper and lower bounds for b, assuming performance occurs.

(c) Assume that the parties split the surplus from renegotiation equally. Derive the resulting value of the bonus b.

6. Consider the following contract model:

V = value of performance to the buyer;
C = fixed cost of performance to the seller;
P = price, payable on performance, where $V>P>C$;
$q(x)$ = probability of performance;
x = seller's effort to avoid breach, where $q'>0$, $q''>0$.

(a) Derive the socially optimal level of effort, x, by the seller.

(b) Show that *expectation damages* induces the seller to choose the socially optimal level of effort.

7. When property can be lost and recovered (e.g., straying cattle, shipwrecks), the economic problems concern incentives to avoid the original loss, and if the loss occurs, to recover the property. Let

y = dollar expenditure by the owner to prevent a loss;
V = value of the property in question;
$q(y)$ = probability of a loss, $q'<0$, $q''>0$.

(a) Assume initially that recovery is not possible. Derive the condition for optimal prevention of loss. How does it vary with V?

(b) Now suppose that the owner can invest in recovery efforts following a loss. Specifically, let

x = dollar expenditure on recovery;
$p(x)$ = probability of a successful recovery, $p'>0$, $p''<0$.

Derive the condition for optimal recovery effort and show how it varies with V. Also, reconsider the optimal investment in loss prevention. How does the possibility of recovery affect the owner's optimal investment in y, if at all? Explain.

(c) Finally, suppose that only someone *other than the owner* can invest in recovery (e.g., a professional treasure hunter). Assuming the same recovery technology, consider two rules for recovery: (1) Recovered property is restored to the original owner, and (2) Recovered property is owned by the finder (a finder-keepers rule). What choice of y will the original owner make, and what choice of x will the treasure hunter make, under each of these rules? Which rule is superior?

8. Consider a two period model of land development and regulation. An owner of undeveloped land can develop *now*, yielding a private benefit of V_1, or he can develop *later*, yielding a private benefit of V_2. Assume that $V_2>V_1$, implying that waiting is privately optimal, all else equal.

Suppose that with probability p, development in either period will cause external damages of D (e.g., loss of the habitat of an endangered species), but with probability $1-p$, no harm will occur. Assume that $D>V_2$, so if the damage will occur, it is efficient to prohibit development. However, the government will only learn whether or not the damage will occur at the start of period two. If it learns that the damage will occur and the land is not yet developed, the government will prohibit development, but if the land was developed in period one, the government will not reverse the outcome.

(a) Show that from a social perspective it is always efficient for the landowner to wait until period two to develop.

(b) Let C be the amount of compensation paid to a landowner who did not develop in period one and is prohibited from developing in period two. Show that $C=0$ will result in an excessive incentive for the landowner to develop in period one. What value of C will induce the landowner to make the efficient decision with respect to the timing of development?

9. Repeat defendants like business firms often hire full time (in-house) attorneys rather than hiring an outside attorney as claims arise. The impact of an in-house attorney is to lower the variable cost of a trial at the expense of imposing fixed costs (the attorney's salary) on the defendant. This exercise asks you to use the asymmetric information model of settlement to derive the strategic effects of lowering the defendant's variable cost of a trial. To do so, modify the model as follows. Let:

αC_d = the defendant's cost of trial, where $\alpha \leq 1$.

All other aspects of the model remain the same.

(a) Derive the first order condition for the optimal settlement amount, $S^*(\alpha)$.

(b) Derive an explicit expression for $S^*(\alpha)$ for the case of the uniform distribution and show that $\partial S^*/\partial\alpha > 0$. Explain the result intuitively.

(c) Also for the uniform distribution, show that the probability of trial is decreasing in α. Explain the result.

(d) Let K be the fixed cost of hiring an in-house attorney. Compare the defendant's expected cost of hiring an in-house attorney ($\alpha<1$, $K>0$) versus an outside attorney ($\alpha=1$, $K=0$) as a function of α.

10. Consider the optimal enforcement model in which only fines are available and the probability of apprehension is variable. Suppose that the gain to potential offenders is distributed uniformly on the interval $[0, \overline{g}]$, the cost of apprehension is $.5p^2$, the harm from an offense is h, and the offender's wealth is w. Given that the optimal fine in this case is $f^*=w$, derive an explicit expression for the optimal probability of apprehension, p^*. Show that p^* is increasing in h, decreasing in $\overline{g}$, and may be increasing or decreasing in w.